Learning Unit 2: Patterns

An essential role in learning mathematics plays the recognition and use of patterns and structures. Patterns and structures often determine individual subject areas and can be used to clarify central basic mathematical ideas.

Pattern Awareness

The notion of pattern awareness implies recognition and expression of the structure of a mathematical pattern (Steffe, 1994). Mathematical patterns are defined as predictable regularities that involve numerical or spatial relationships (Mulligan & Mitchelmore, 2009). The development of pattern awareness is based on understanding the concept of "unit of repeat" and "composite unit"; these are important for children's future experiences with (a) measurement that involves spatial repeating units and (b) multiplication that involves numerical repeating units (Papic et al., 2011). Previous research showed that pattern awareness is an index for children's ability to formulate generalizations, while it is strongly associated with the processes of representing, symbolizing, and abstracting mathematical relationships (Liljedahl, 2004; Mulligan & Mitchelmore, 2009). Children's abilities in recognizing and expressing patterns have also been related with their abilities in analogical and inductive reasoning (English, 2004).

Definition: Repeating and Growing Patterns

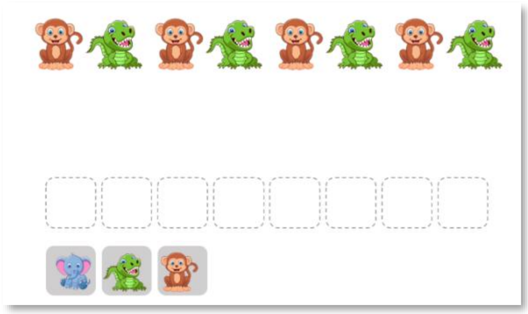
Mathematical patterns may have the structure of a *repeating pattern* or a *growing pattern*.

- *Repeating patterns* are characterized by a "a cyclic structure" that is established by the repeated application of a smaller part of the pattern (Zazkis & Liljedahl, 2002; Liljedahl, 2004). For example, the pattern ABABAB has a minimum unit of repeat of length two, while the pattern ABBABBABB has a minimal unit of repeat of length three (Tsamir et al., 2020). The concept of unit of repeat develops as children discuss regularity in repeating patterns that may include motion, color, sound, position, and quantity, and as they get engaged with describing, extending, transferring, translating, and creating patterns (Economopoulos, 1998). Children usually experience first repeating patterns that involve a simple repetition of a single element (e.g., ABABAB) (Steffe, 1994).

- *Growing patterns* are characterized by a sequence of elements that increase systematically (e.g., the sequence of even numbers) (Papic et al., 2011). Growing patterns have been related to the development of early algebraic thinking abilities and considered as appropriate for the early years of primary school (Warren & Cooper, 2008).

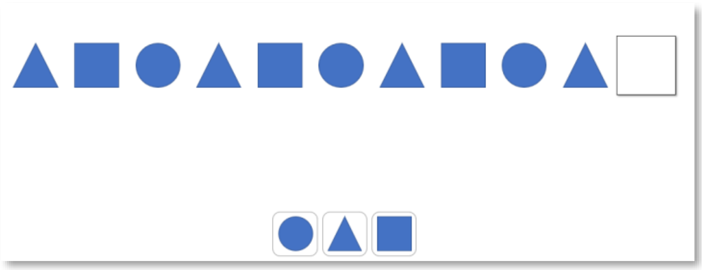
Pattern Activities and Tasks to Develop Pattern Awareness

Tasks with patterns may involve copying a pattern which means making an exact copy of the pattern (Rittle-Johnson et al., 2013), using the same materials (example 1).



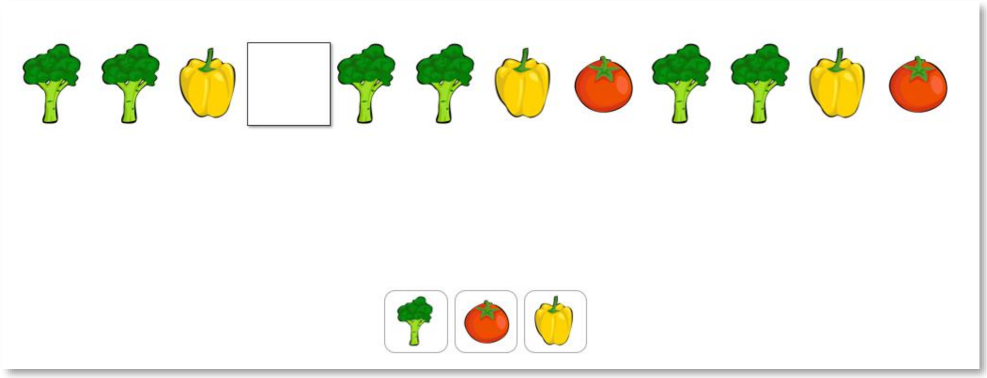
Example 1

Extending patterns, which is also a very important skill, means that the children know which element in a pattern comes next (Example 2).



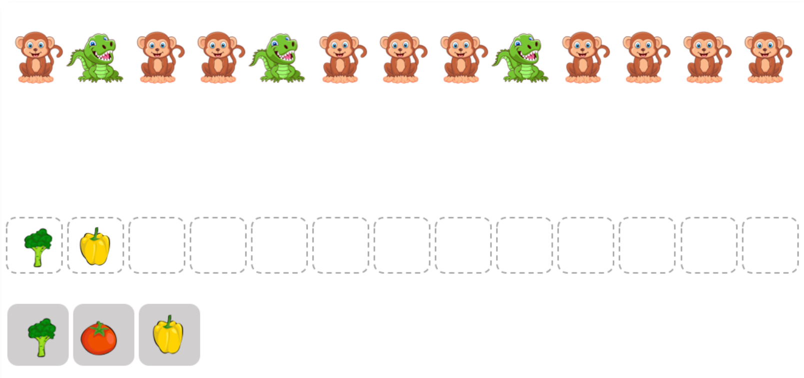
Example 2

Repairing patterns means that the children know how to correct a wrong or incomplete pattern (Example 3). So, they must complete a missing shape in a pattern.



Example 3

Translating patterns is a different type of task which requires children to copy a pattern, using the same structure but different materials, i.e., copying a pattern that is represented by block to a pattern that is represented by tiles or drawing the patterns with colored pencils (Papic et al., 2011) (Example 4). They must create and identify a pattern that follows the same rule as a given pattern. Translating patterns have been found to be more complex than for example copying patterns (Rittle-Johnson et al., 2013; Sarama & Clements, 2009).



Example 4

Moreover, there are tasks that differ with respect to the level of complexity. For example, the easiest task is to copy a pattern that contains a total of two elements (AB), then a pattern where there are four elements (ABAB), and then six elements (ABABAB) (Papic et al., 2011). Sarama and Clements (2009) also showed that there is developmental progression for pattern and structure; by the age of 4, children are able to copy a pattern with an AB structure and ABB structure, whereas at the age of 6 years, they can translate these patterns using different materials.

Since pattern and structure awareness are so important, here are a few ideas on how to support and encourage them. Pattern and structure awareness may be promoted, for example through the following set of indicative tasks:

- Copy patterns (for example AB, ABB, and ABC), represented with pictures, shapes (different color and sizes), and numbers.
- Extend AB patterns (for example AB, ABB, and ABC), represented with pictures, shapes (different color and sizes), and numbers.
- Repair patterns (for example AB, ABB, and ABC), represented with pictures, shapes (different color and sizes), and numbers.
- Translate patterns (for example AB, ABB, ABC, and AABB), represented with pictures.
- Copy, extend, and translate more complex patterns (for example ABCD, ABBC, AABC).

Strategies

In literature, different strategies are distinguished in dealing with patterns.

A typical strategy that children use when they copy patterns is the "*direct comparison*" strategy, where they match one-by-one object at a time (Papic et al., 2011). A second strategy, known as "*alternation" strategy*, is evident when students focus and describe the way the objects alternate, i.e., saying, "first blue, second yellow, then blue, then yellow…" (Papic et al., 2011). A third strategy is the *"unit strategy"* which is evident when children recognize the unit of repeat and they do not comment on the position in the sequence, but they express the unit of repeat, i.e., blue-yellow, blue-yellow, emphasizing that these two objects form a single unit. (Tsamir et al., 2020; Lüken & Sauzet, 2020).

Difficulties

Previous research has reported various difficulties that children face when, for example, they copy or translate repeating patterns. These relate to the number of the elements presented in the model pattern (Tsamir et al., 2020). For example, children may copy the pattern ABABABAB as ABABABABBBBBB, probably because there are extra materials available to them and not the exact number (Rittle-Johnson et al., 2013). Students may notice singular characteristics of a pattern (e.g., color, shape, number), without being able to express a regularity (Lüken & Sauzet, 2020). Another difficulty is the identification of the starting point of the pattern. For example, when children are asked to copy a pattern like GGRGGRGGRGGR, they may produce a pattern like RGGRGGRGGRGG, indicating that they read the pattern from right to left rather than left to right (Warren, 2005). Generally, children have less difficulties in copying patterns compared to extending patterns; naming the last element of a pattern, translating, and identifying the repeating pattern are the most difficult tasks (Lüken & Sauzet, 2020).

How the MADITA app supports the concept of patterns becomes evident in the learning unit that introduces the app.

The next learning unit focuses on the topic of *Numbers*.

References

Economopoulos, K. (1998). What comes next? The mathematics of pattern in kindergarten. *Teaching Children Mathematics, 5*(4), 230–234.

English, L. D. (2004). Mathematical and analogical reasoning in early childhood. In L.D. English, (Ed.) *Mathematical and analogical reasoning of young learners*. Routledge.

Liljedahl, P. (2004). Repeating pattern or number pattern: The distinction is blurred. *Focus on learning problems in mathematics*, *26*(3), 24-42.

Lüken, M. M., & Sauzet, O. (2020). Patterning strategies in early childhood: a mixed methods study examining 3-to 5-year-old children's patterning competencies. *Mathematical Thinking and Learning, 23*(1), 28-48.

Mulligan, J., & Mitchelmore, M. (2009). Awareness of pattern and structure in early mathematical development. *Mathematics Education Research Journal, 21*(2), 33–49. https://doi.org/10.1007/BF03217544

Papic, M. M., Mulligan, J. T., & Mitchelmore, M. C. (2011). Assessing the Development of Preschoolers' Mathematical Patterning. *Journal for Research in Mathematics Education, 42*(3), 237–268. https://doi.org/10.5951/jresematheduc.42.3.0237

Rittle-Johnson, B., Fyfe, E. R., McLean, L. E., & McEldoon, K. L. (2013). Emerging understanding of patterning in 4-year-olds. *Journal of cognition and development*, *14*(3), 376-396.

Sarama, J., & Clements, D. H. (2009). *Early childhood mathematics education research: Learning trajectories for young children*. Routledge.

Steffe, L. (1994). Children's multiplying schemes. In G. Harel & J. Confrey (Eds.), *The development of multiplicative reasoning in the learning of mathematics* (pp. 3-41). State University of New York Press.

Tsamir, P., Tirosh, D., Barkai, R., Levenson, E. (2020). Copying and Comparing Repeating Patterns: Children's Strategies and Descriptions. In M. Carlsen, I. Erfjord & P.S.

Hundeland (Eds.), *Mathematics Education in the Early Years.* Springer, Cham. https://doi.org/10.1007/978-3-030-34776-5_4

Warren, E., & Cooper, T. (2003). Introducing equivalence and inequivalence in Year 2. *Australian Primary Mathematics Classroom, 8*(10), 4-9.

Warren, E. (2005). Young Children's Ability to Generalise the Pattern Rule for Growing Patterns. *International Group for the Psychology of Mathematics Education*, *4*, 305-312.

Zazkis, R., & Liljedahl, P. (2002). Generalization of patterns: The tension between algebraic thinking and algebraic notation. *Educational studies in mathematics, 49*, 379-402.